Spectral Independence & Field Dynamics

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Glauber dynamics on distribution μ over $\Omega \subseteq \{0,1\}^V$

Start from an arbitrary configuration $X \in \Omega$. In t-th transition step,

- Pick a variable $v \in V$ uniformly at random
- Resample X_v from μ conditional on X_{V-v}

Influence Matrix: $\Psi_\mu(u,v) = \mu(\sigma_v = 1 \mid \sigma_u = 1) - \mu(\sigma_v = 1 \mid \sigma_u = 0)$

η-**Spectral Independence**: for any $\Lambda \subseteq V$, and any $\tau \in \{0,1\}^\Lambda$

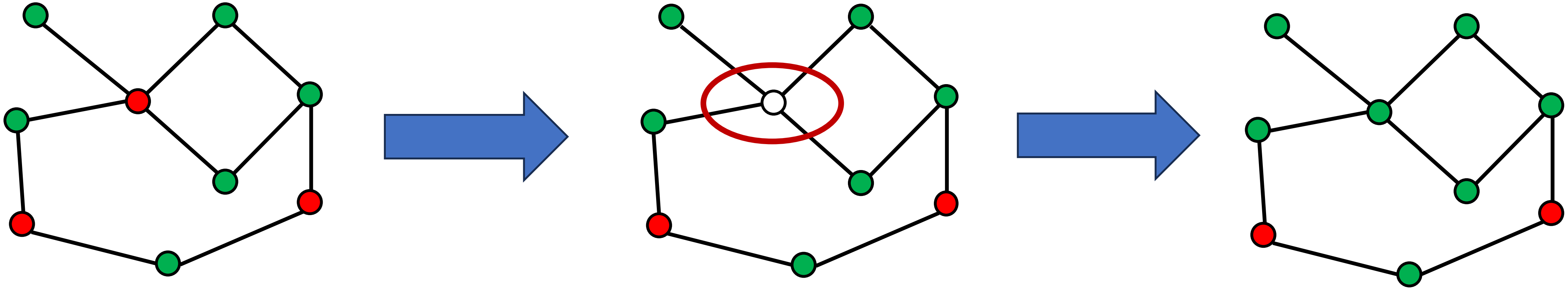
$$\lambda_{\max}\left(\Psi_{\mu^\tau}\right) \leq \eta$$

Theorem [Anari, Liu, Oveis Gharan 2020]

η- Spectral Independence $\Rightarrow$ $\frac{1}{n^{O(\eta)}}$ spectral gap of Glauber dynamics

Glauber dynamics and down up walk

Glauber Update = Down-Walk + Up-Walk



- pick one vertex $v \in V$ u.a.r
- Remove the value at v

Down Walk $D_{n \to n-1}$

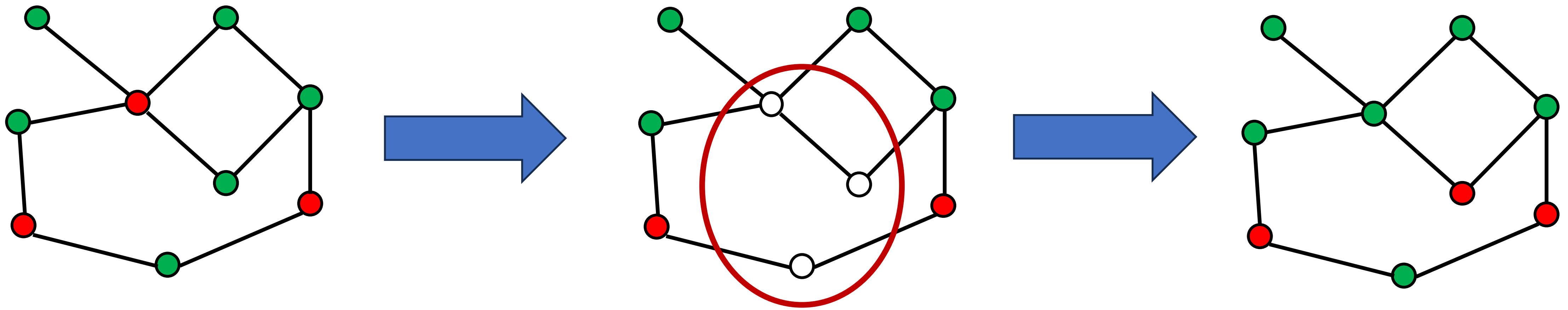
- Resample the value at “white” variable
- $X_v \sim \mu_v(\cdot \mid X_{V-v})$

Up Walk $U_{n-1 \to n}$

Glauber dynamics and block dynamics

Glauber dynamics: update **one variable** in each step

Block dynamics: update ***k* variables** in each step



- pick $S \subseteq V$ with $|S| = k$ u.a.r.
- Remove all the values at S

- Resample the value at "white" variables
- $X_S \sim \mu_S(\cdot \mid X_{V-S})$

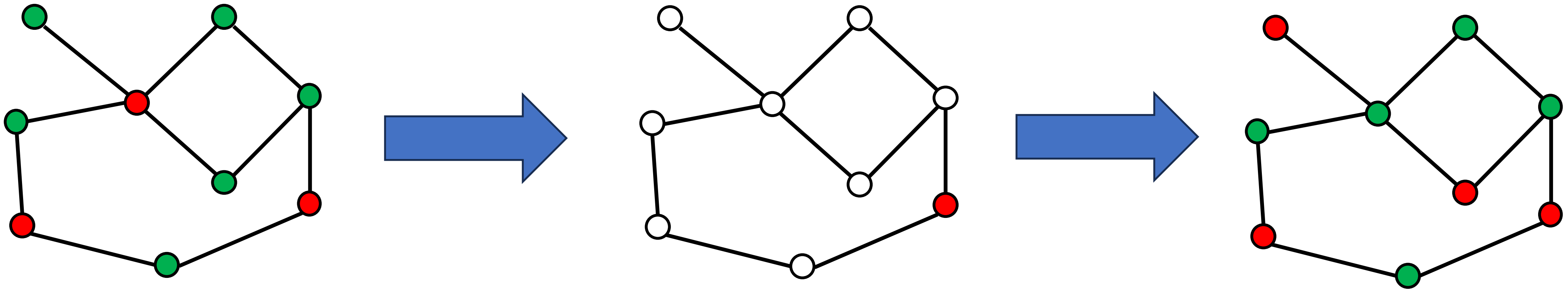
Down Walk $D_{n \to n-k}$

Up Walk $U_{n-k \to n}$

An extremal case of block dynamics

Block dynamics: $P = D_{n\to 1}U_{1\to n}$

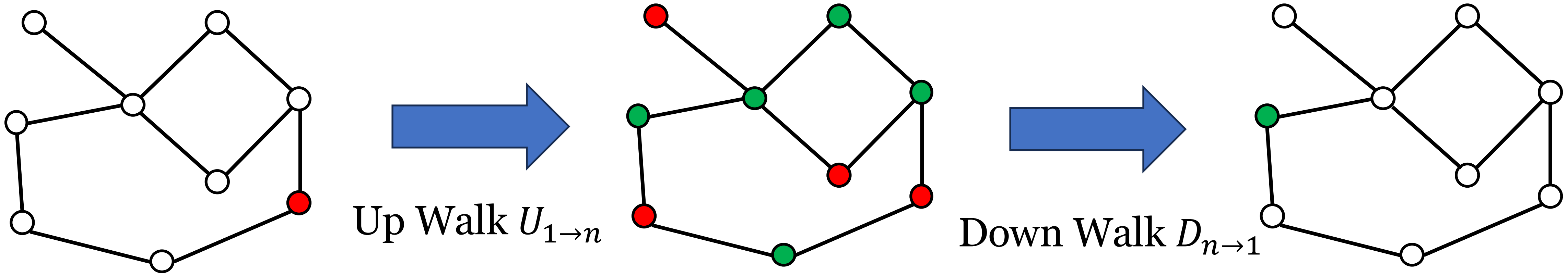
- Pick $n-1$ vertices $S \subseteq V$ uniformly at random and let $u = V - S$
- Resample X_S from μ conditional on X_u



Lemma : $\lambda_2(D_{n\to 1}U_{1\to n}) = \frac{1}{n}\lambda_{\max}(\Psi_\mu)$

Form down-up to up-down walk

Fact: $\lambda_2(D_{n\to1}U_{1\to n}) = \lambda_2(U_{1\to n}D_{n\to1})$ What is the random walk $U_{1\to n}D_{n\to1}$?



Up-Down Walk: each state is a ***pair*** $(u, c) \in V \times \{0,1\}$

- Pick a vertex $v \in V$ uniformly at random
- Sample c' from the marginal distribution at v conditional on $X_u = c$
- Update $(u, c) \to (v, c')$

$$\begin{array}{cc} & (v,c') \\ (u,c) & \begin{bmatrix} & & \\ & \frac{1}{n}\Pr_{\mu}[X_v = c' \mid X_u = c] & \\ & & \end{bmatrix} \end{array} = \boxed{U_{n\to 1} D_{1\to n}}$$

Stationary distribution of the up-down walk: $\mu D_{1\to n}$

For every pair (u, c), $\text{Stationary}(u, c) = \frac{1}{n}\Pr_{\mu}[X_u = c]$

$$\lambda_{max}\ \begin{array}{cc} & (v,c') \\ (u,c) & \left[\begin{array}{c} \frac{1}{n}\Pr_{\mu}[X_v = c' \mid X_u = c] - \frac{1}{n}\Pr_{\mu}[X_v = c'] \\ \\ \approx \frac{1}{n}\Psi_\mu \end{array} \right] \end{array} = \boxed{\lambda_2(U_{n\to 1}D_{1\to n})}$$

$$\Rightarrow \quad \lambda_2(D_{n\to 1}U_{1\to n}) = \lambda_2(U_{1\to n}D_{n\to 1}) = \frac{1}{n}\lambda_{\max}(\Psi_\mu)$$

Dirichlet form of Glauber dynamics can be written as

$$\mathcal{E}_{\mu,P}(f,f) = \frac{1}{|V|}\sum_{v\in V} \mathbf{E}_\mu\left[\mathbf{Var}_\mu(f \mid X_{-v})\right]$$

Dirichlet form of **block dynamics** (update k variables) can be written as

$$\mathcal{E}_k(f,f) = \frac{1}{\binom{n}{k}}\sum_{|S|=k} \mathbf{E}_\mu\left[\mathbf{Var}_\mu(f \mid X_{V\setminus S})\right]$$

Spectral independence implies spectral gap of the down-up walk $D_{n\to 1}U_{1\to n}$:

$$\mathbf{Var}_\mu(f) \le \left(1-\frac{\eta}{n}\right)^{-1} \mathcal{E}_{n-1}(f,f)$$

Spectral gap of the down-up walk $D_{n\to 1}U_{1\to n}$:

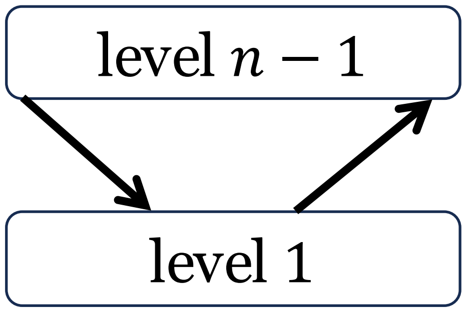
$$\mathbf{Var}_\mu(f) \le \left(1-\frac{\eta}{n}\right)^{-1} \mathcal{E}_{n-1}(f,f)$$

By the definition of Dirichlet form

$$\mathbf{Var}_\mu(f) \le \left(1-\frac{\eta}{n}\right)^{-1} \frac{1}{\binom{n}{n-1}} \sum_{|S|=n-1} \mathbf{E}_\mu\left[\mathbf{Var}_\mu(f \mid X_{V\setminus S})\right]$$

Apply spectral independence again for $\underbrace{\mu \text{ condition on } X_{V\setminus S}}_{\pi:\ n-1 \text{ free variables in } S}$.

down-up walk on π



$$\mathbf{Var}_\pi(f) \le \left(1-\frac{\eta}{n-1}\right)^{-1} \frac{1}{\binom{n-1}{n-2}} \sum_{|T|=n-2} \mathbf{E}_\pi\left[\mathbf{Var}_\pi(f \mid X_{S\setminus T})\right]$$

Spectral gap of the down-up walk $D_{n\to 1}U_{1\to n}$:

$$\mathbf{Var}_\mu(f) \le \left(1-\frac{\eta}{n}\right)^{-1} \mathcal{E}_{n-1}(f,f)$$

By the definition of Dirichlet form

$$\mathbf{Var}_\mu(f) \le \left(1-\frac{\eta}{n}\right)^{-1} \frac{1}{\binom{n}{n-1}} \sum_{|S|=n-1} \mathbf{E}_\mu\left[\mathbf{Var}_\mu(f \mid X_{V\setminus S})\right]$$

Apply spectral independence again for $\underbrace{\mu \text{ condition on } X_{V\setminus S}}_{\pi:\, n-1 \text{ free variables in } S}$.

$$\mathbf{Var}_\mu(f) \le \left(1-\frac{\eta}{n}\right)^{-1}\left(1-\frac{\eta}{n-1}\right)^{-1} \frac{1}{\binom{n}{n-2}} \sum_{|S|=n-2} \mathbf{E}_\mu\left[\mathbf{Var}_\mu(f \mid X_{V-S})\right]$$

Spectral gap of the down-up walk $D_{n\to 1}U_{1\to n}$:

$$\mathbf{Var}_\mu(f) \le \left(1-\frac{\eta}{n}\right)^{-1} \mathcal{E}_{n-1}(f,f)$$

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$$\mathbf{Var}_\mu(f) \le \left(1-\frac{\eta}{n}\right)^{-1} \frac{1}{\binom{n}{n-1}} \sum_{|S|=n-1} \mathbf{E}_\mu\left[\mathbf{Var}_\mu(f \mid X_{V\setminus S})\right]$$

Repeating this process until the size of S becomes 1

$$\mathbf{Var}_\mu(f) \le \prod_{i=2}^{n} \left(1-\frac{\eta}{i}\right)^{-1} \frac{1}{n} \sum_{v\in V} \mathbf{E}_\mu\left[\mathbf{Var}_\mu(f \mid X_{-v})\right]$$

Approximate tensorization of variance $\Longleftrightarrow$ Spectral gap of Glauber dynamics

Repeating this process until the size of S becomes $1 \le \ell < n$

$$\mathbf{Var}_\mu(f) \le \prod_{i=\ell+1}^{n} \left(1 - \frac{\eta}{i}\right)^{-1} \frac{1}{\binom{n}{\ell}} \sum_{|S|=\ell} \mathbf{E}_\mu \left[\mathbf{Var}_\mu(f \mid X_{V-S})\right]$$

Approximate factorization of variance $\Longleftrightarrow$ Spectral gap of block dynamics

Lemma: the spectral gap of ℓ-block dynamics updating ℓ variables is

$$\prod_{i=\ell+1}^{n} \left(1 - \frac{\eta}{i}\right) \approx exp\left(-\sum_{i=\ell+1}^{n} \frac{\eta}{i}\right) \approx \left(\frac{\ell}{n}\right)^{O(\eta)}$$

Corollary: the spectral gap of Glauber dynamics ($\ell = 1$) is $\left(\frac{1}{n}\right)^{O(\eta)}$

References

Theorem [Anari, Liu, Oveis Gharan 2020]

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Other Proofs

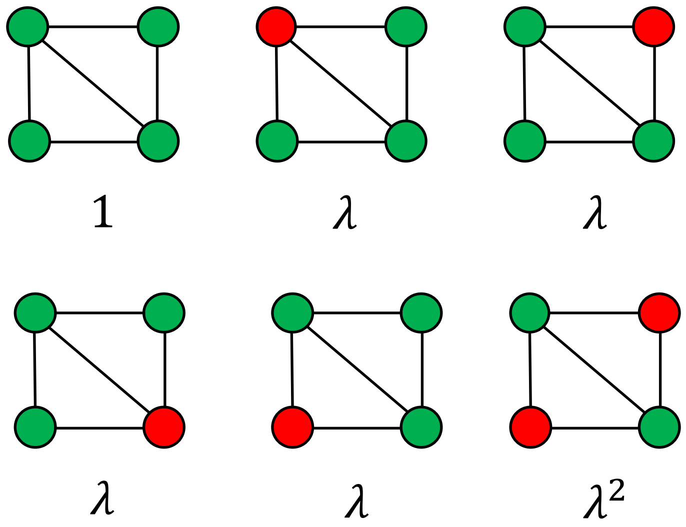
- Proof based on **high-dimensional expanders** [Alev and Lau, ArXiv: 2001.02827]
- Proof based on **martingale**
  - Cryan, Guo, and Mousa, ArXiv: 1903.06081
  - Anari, Jain, Koehler, Pham, Vuong, ArXiv: 1903.06081 Appendix B

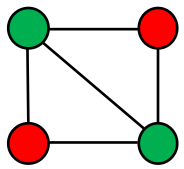
Hardcore model

- Graph $G = (V, E)$: n-vertex and max degree Δ;
- Fugacity parameter $\lambda \in \mathbb{R}_{\geq 0}$;
- Configuration $X \in \{0,1\}^V$
  - $X_v = 0$: vertex v is **occupied**
  - $X_v = 1$: vertex v is **unoccupied**
- $X \in \Omega$ if occupied vertices form an independent set
- Gibbs distribution μ:

$$\forall X \in \Omega, \qquad \mu(X) \propto w(X) = \lambda^{|X|_1}.$$

$|X|_1 =$ *number of occupied vertices* $(X_v = 1)$



$\mu\Bigg($  $\Bigg) = \dfrac{\lambda^2}{1 + 4\lambda + \lambda^2}$

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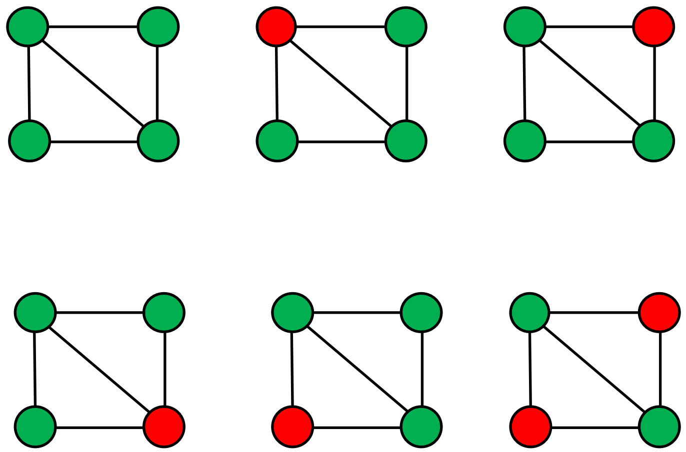


Uniqueness Threshold

$$\lambda_c(\Delta) = \frac{(\Delta-1)^{(\Delta-1)}}{(\Delta-2)^{\Delta}} \approx \frac{e}{\Delta}$$

Δ: maximum degree

Spectral independence of hardcore model

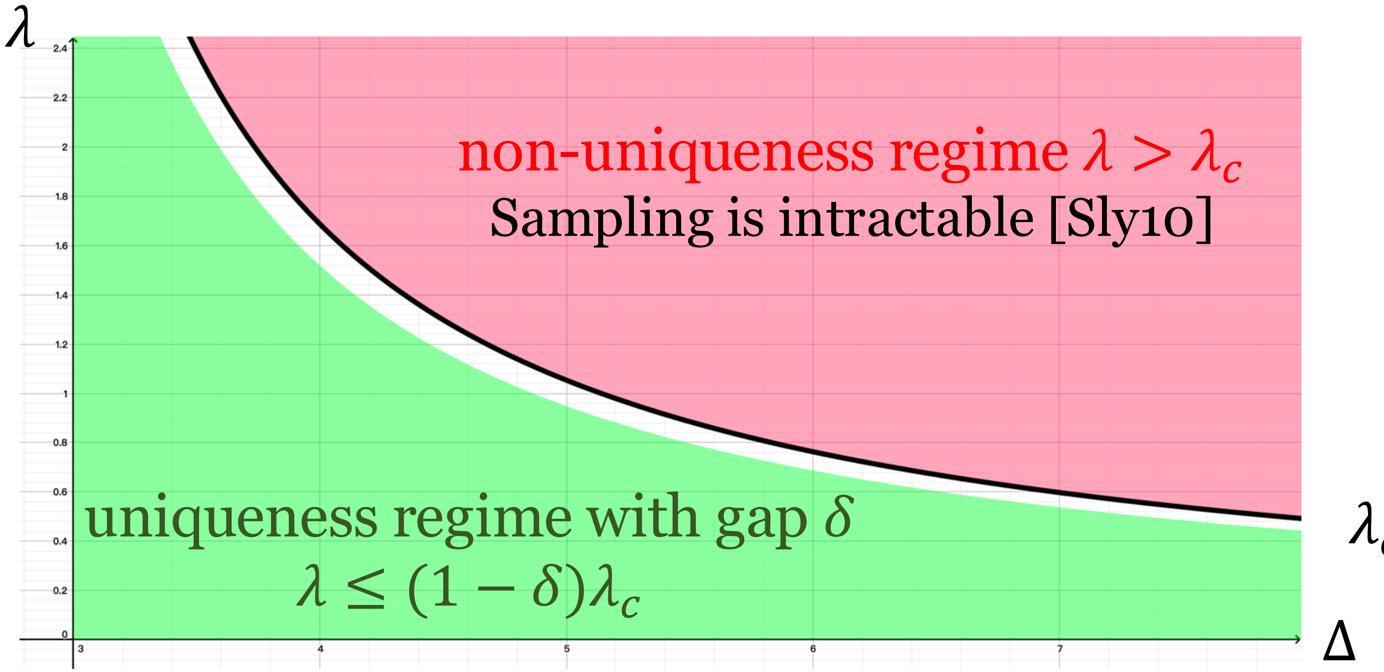
$$\lambda \leq (1-\delta)\lambda_c = (1-\delta)\frac{(\Delta-1)^{(\Delta-1)}}{(\Delta-2)^{\Delta}}$$

$\Rightarrow$

$O\left(\frac{1}{\delta}\right)$-spectral independence

Theorem [Anari, Liu, Oveis Gharan 2020]

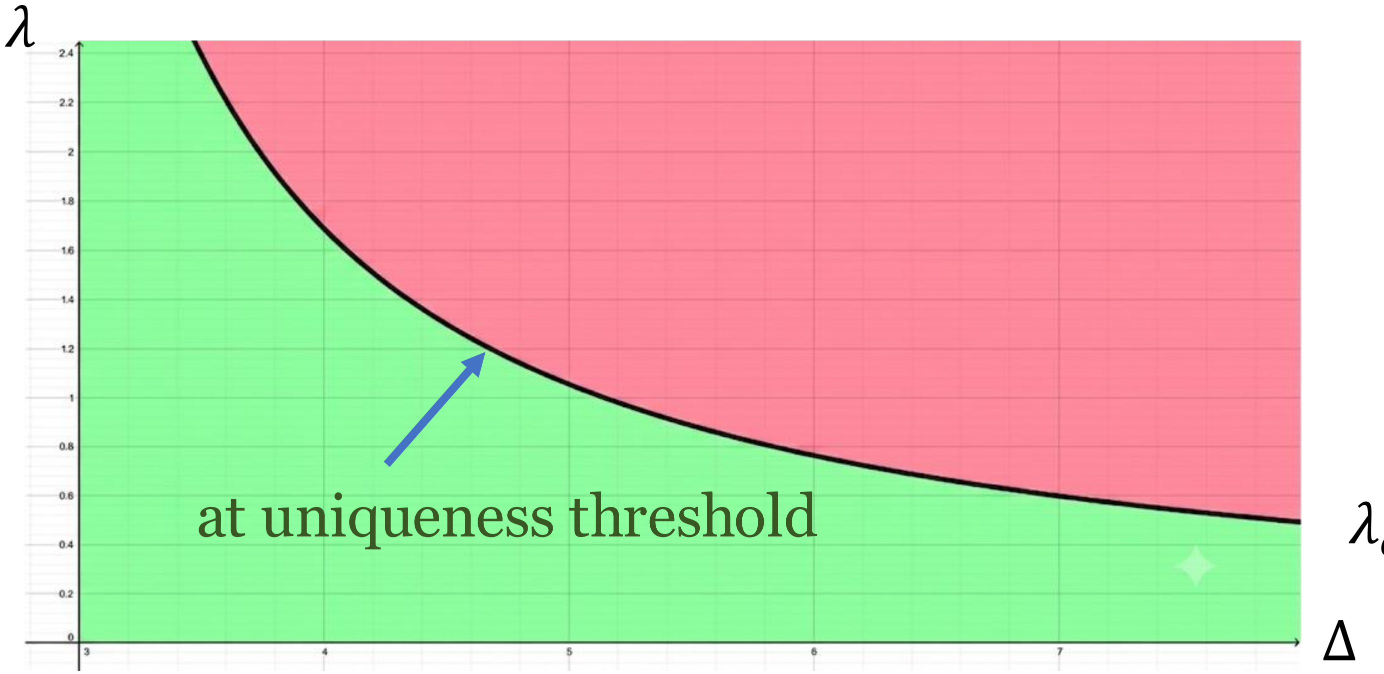
If $\lambda \leq (1-\delta)\lambda_c$, then the ***spectral gap*** if Glauber dynamics is $\frac{1}{n^{O(1/\delta)}}$ and the ***mixing time*** is $\tau_{\mathrm{mix}}\left(\frac{1}{2e}\right) = n^{O(1/\delta)}$



$$\lambda_c(\Delta) = \frac{(\Delta - 1)^{(\Delta-1)}}{(\Delta - 2)^{\Delta}}$$

Problem : *fixed parameter trackable* sampling algorithm for hardcore model

Let $\delta > 0$ be an *arbitrary gap*. For any hardcore model with $\lambda \leq (1-\delta)\lambda_c(\Delta)$, can we sample from Gibbs distribution in time $C(\delta) \cdot \mathrm{poly}(n)$?



$$\lambda_c(\Delta) = \frac{(\Delta - 1)^{(\Delta - 1)}}{(\Delta - 2)^{\Delta}}$$

Problem : mixing **at the uniqueness threshold**.

Can we sample from the Gibbs distribution μ in **polynomial time** when

$$\lambda = \lambda_c(\Delta) = \frac{(\Delta - 1)^{(\Delta - 1)}}{(\Delta - 2)^{\Delta}}$$

Mixing time of Glauber dynamics when $\lambda \le (1-\delta)\lambda_c$

Work	Mixing Time	Technique
Anari, Liu, Oveis Gharan, 2020 improved by Chen, Liu, Vigoda, 2020	$n^{O(1/\delta)}$	Spectral Independence (SI)
Chen, Liu, Vigoda, 2021	$\Delta^{O(\Delta^2/\delta)} n \log n$	
Chen, F., Yin, Zhang, 2021	$e^{O(1/\delta)} n^2 \log n$	SI & Field Dynamics

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Mixing time of Glauber dynamics when $\lambda = \lambda_c$ [Chen, Chen, Yin, Zhang, 2025]

poly(n) mixing time proved via **Field Dynamics**

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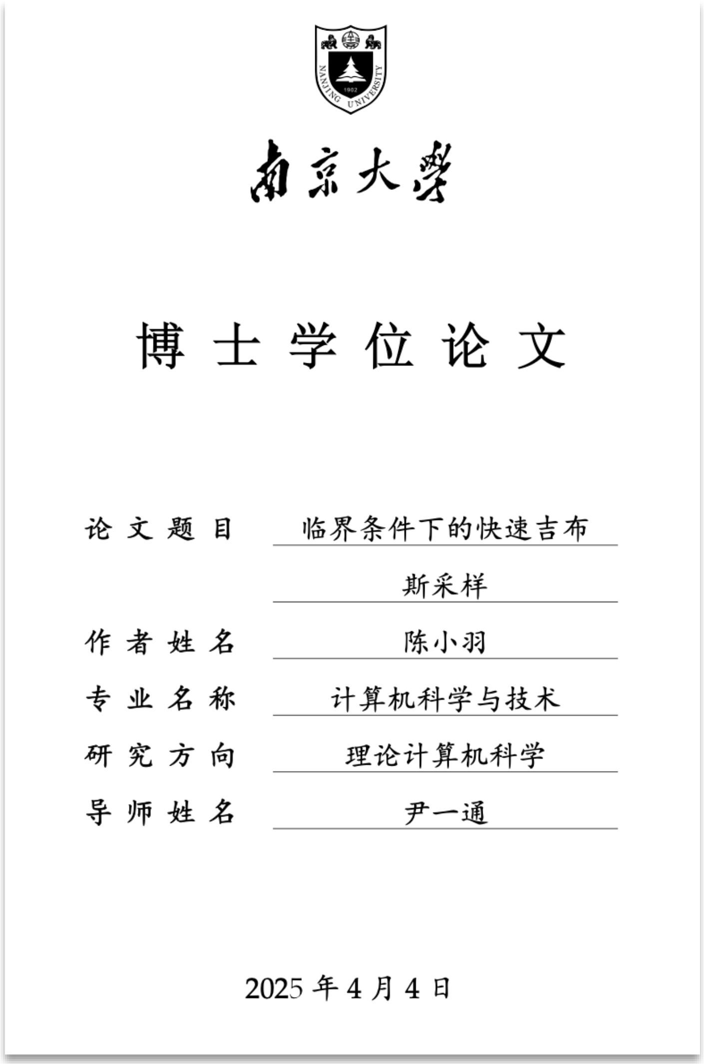
New Markov chain: *field dynamics*


南京大學

博 士 学 位 论 文

论 文 题 目　临界条件下的快速吉布斯采样

作 者 姓 名　陈小羽

专 业 名 称　计算机科学与技术

研 究 方 向　理论计算机科学

导 师 姓 名　尹一通

2025 年 4 月 4 日

- 场动力系统的定义看起来像是灵光一现。实际上这个定义的发现背后经历了漫长的思考。这将回答

  “**你们是如何想到这个定义的**？”

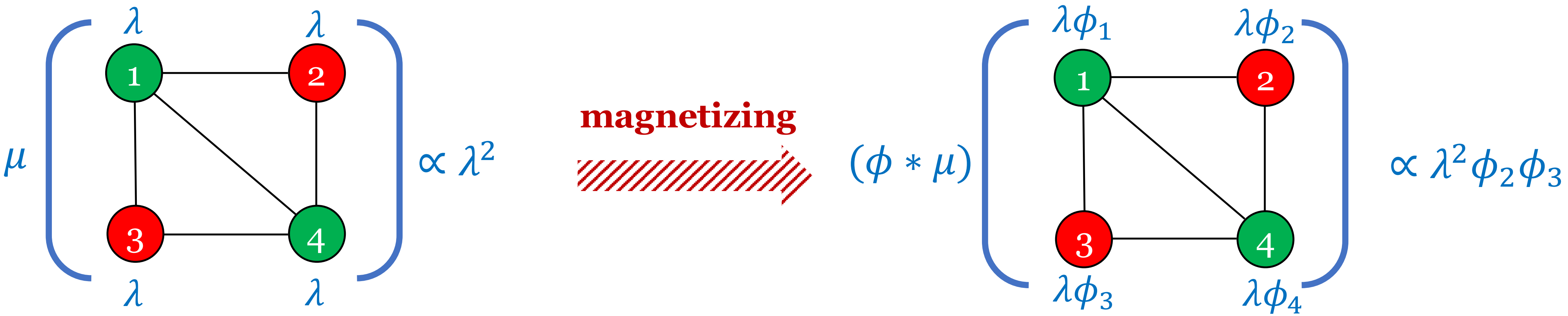
- 场动力系统目前有**三种**不同的理解方式。或者说, 站在此前的工作的基础上,有三个不同的想法都能把人们引导至场动力系统。

https://www.ccf.org.cn/Awards/Awards/2025-11-20/853327.shtml

Magnetizing joint distribution with local fields

Joint distribution μ over $\{0,1\}^V$, local fields $\phi = (\phi_v)_{v \in V} \in \mathbb{R}^V_{>0}$

$$(\boldsymbol{\phi} * \mu)(\sigma) \propto \mu(\sigma) \prod_{v \in V : \sigma_v = 1} \phi_v$$



Hardcore model: $\mu(S) \propto \lambda^{|S|}$

Hardcore mode with local fields
$\mu^{(\phi)}(S) \propto \lambda^{|S|} \prod_{v \in S} \phi_v = \prod_{v \in S} \lambda \phi_v$

Field Dynamics

Input: a distribution μ over $\{0,1\}^V$, a parameter $\theta \in (0,1)$

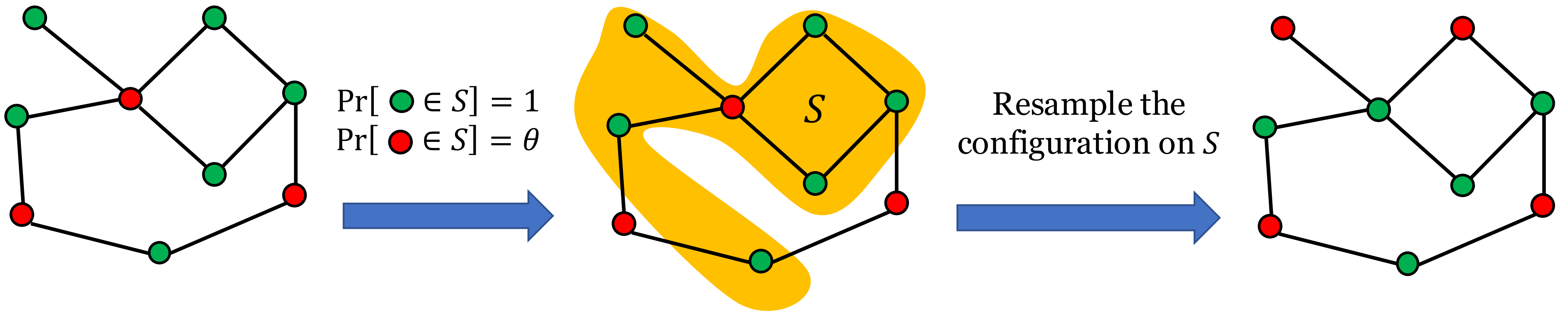
Start from an arbitrary feasible configuration $X \in \{0,1\}^V$

For each t from 1 to T **do**

- Construct $S \subseteq V$ be selecting each $v \in V$ independently with probability

$$p_v = \begin{cases} 1 & \text{if } X_v = 0 \\ \theta & \text{if } X_v = 1 \end{cases}$$

- Resample $X_S \sim (\boldsymbol{\theta} * \mu)_S(\cdot \mid X_{V \setminus S})$ conditional distribution induced from $(\boldsymbol{\theta} * \mu)$



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Proposition (Field Dynamics): for any $\theta \in (0,1)$

The Field Dynamics $P_{FD}(\theta)$ is irreducible, aperiodic and reversible with respect to μ.

$P_{FD}(\theta)$ has the unique stationary distribution μ.

Comparison lemma of spectral gap [Chen, Feng, Yin, Zhang, 2021]

For any distribution μ over $\{0,1\}^V$

$$\lambda_{\text{gap}}^{GD}(\mu) \geq \lambda_{\text{gap}}^{\text{Field}}(\mu,\theta) \cdot \lambda_{\text{mingap}}^{\text{GD}}(\boldsymbol{\theta} * \mu), \qquad \boldsymbol{\theta}_v = \theta \text{ for all } v \in V$$

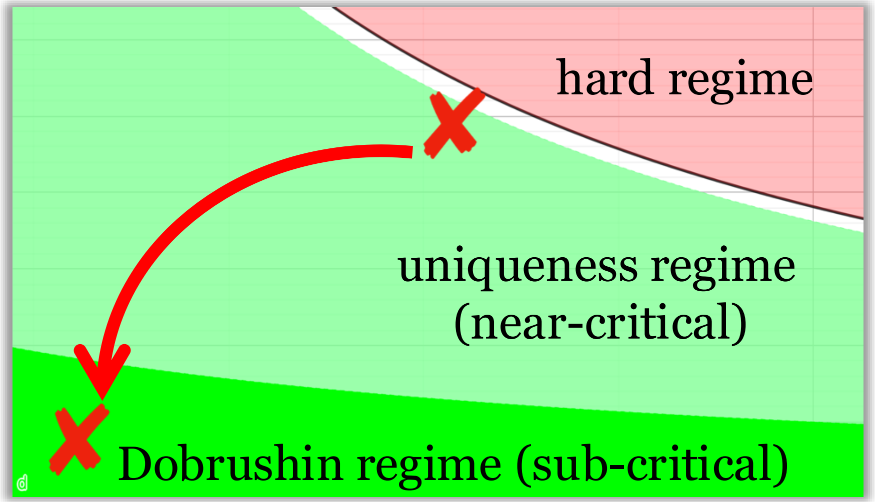
$\lambda_{\text{mingap}}^{\text{GD}}(\boldsymbol{\theta} * \mu)$: *minimum spectral gap of Glauber dynamics for all conditional distributions induced by* $\boldsymbol{\theta} * \mu$.

$\theta * \mu \Longleftrightarrow$ hardcore model with $\theta\lambda$

$\theta = \dfrac{1}{10} \Longrightarrow \theta\lambda = \dfrac{\lambda}{10} \ll \lambda_c$

$\Downarrow$

$\lambda_{\text{mingap}}^{\text{GD}}(\boldsymbol{\theta} * \mu) = \Omega\left(\frac{1}{n}\right)$ [Bubley and Dyer, Path coupling, 1997]



Complete spectral independence

Complete Spectral independence [Chen, Feng, Yin, Zhang, 2021]

There are constant $\eta > 0$ s.t.

for all local fields $\phi \in (0,1]^V$ (for all $v \in V$, $0 < \phi_v \le 1$),

$(\boldsymbol{\phi} * \mu)$ is *spectrally independent* with parameter η

spectral independence: spectral radius $\rho\left(\left|\Psi_{(\phi*\mu)}\right|\right) \le \eta$

Example: hardcore model (G, λ) is ***completely spectrally independent*** if

any hardcore models $(G, (\lambda_v)_{v\in V})$ with $\lambda_v \le \lambda$

are ***spectrally independent***

Complete spectral independence

Complete Spectral independence [Chen, Feng, Yin, Zhang, 2021]

There are constant $\eta > 0$ s.t.

for all local fields $\phi \in (0,1]^V$ (for all $v \in V$, $0 < \phi_v \le 1$),

$(\boldsymbol{\phi} * \mu)$ is *spectrally independent* with parameter η

spectral independence: spectral radius $\rho\left(\left|\Psi_{(\phi*\mu)}\right|\right) \le \eta$

Theorem [Chen, Feng, Yin, Zhang, 2021]

Hardcore model is ***completely spectrally independent*** with $\eta = O(1/\delta)$ if

$$\lambda \le (1-\delta)\lambda_c(\Delta) = (1-\delta)\frac{(\Delta-1)^{(\Delta-1)}}{(\Delta-2)^{\Delta}}$$

Comparison lemma of spectral gap [Chen, Feng, Yin, Zhang, 2021]

For any distribution μ over $\{0,1\}^V$

$$\lambda_{\text{gap}}^{GD}(\mu) \geq \lambda_{\text{gap}}^{\text{Field}}(\mu, \theta) \cdot \lambda_{\text{mingap}}^{\text{GD}}(\boldsymbol{\theta} * \mu), \qquad \boldsymbol{\theta}_v = \theta \text{ for all } v \in V$$

Spectral gap of field dynamics

If μ is η-completely spectrally independent, for any $\theta \in (0,1)$

$$\lambda_{\text{gap}}^{\text{Field}}(\mu, \theta) \geq \theta^{O(\eta)}$$

For hardcore model with $\lambda < (1-\delta)\lambda_c$, take $\theta = 1/10$

$$\lambda_{\text{gap}}^{\text{GD}}(\mu) \geq \theta^{O\left(\frac{1}{\delta}\right)} \cdot \lambda_{\text{mingap}}^{\text{GD}}(\boldsymbol{\theta} * \mu) = \Omega_\delta\left(\frac{1}{n}\right) \Longrightarrow T_{mix} = O(n^2 \log n)$$

μ over $\{0,1\}^V$
integer $k \geq 1$

k-transformation $\longrightarrow$

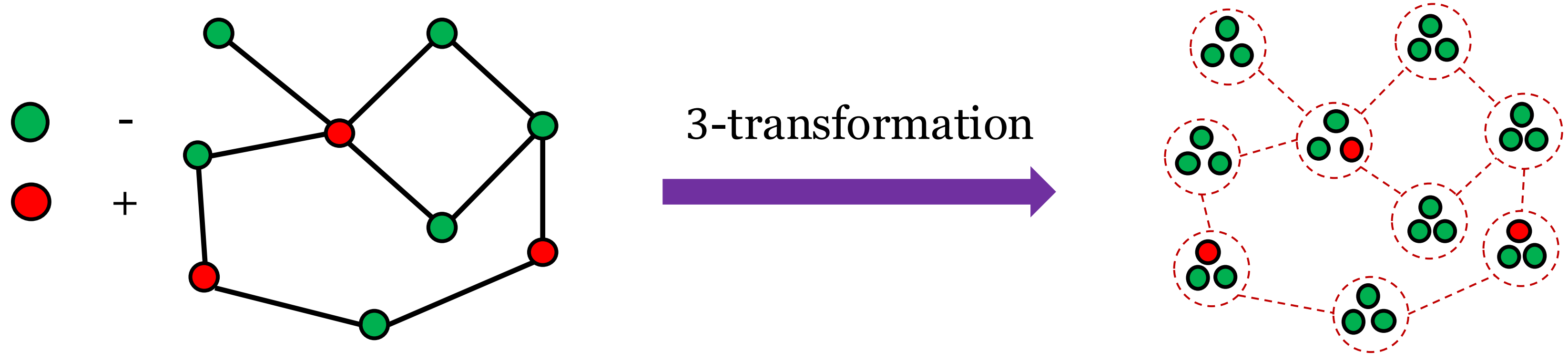
μ_k over $\{0,1\}^{V_k}$
$V_k = \{u_1, u_2, \ldots, u_k \mid u \in V\}$

$X \sim \mu$ $\longrightarrow$

For each variable $u \in V$ **do**

- **If** $X(u) = 0$, **then** $Y(u_i) = 0$ for all $i \in [k]$;
- **If** $X(u) = 1$, **then**
  - Sample $j \in \{1,2,..,k\}$ uniformly at random;
  - $Y(u_j) = 1$ and $Y(u_i) = 0$ for all $i \in [k] \setminus \{j\}$;

$\longrightarrow$ $Y \sim \mu_k$



$X \sim \mu$ $\qquad$ $Y \sim \mu_3$

Original distribution
μ over $\{0,1\}^V$

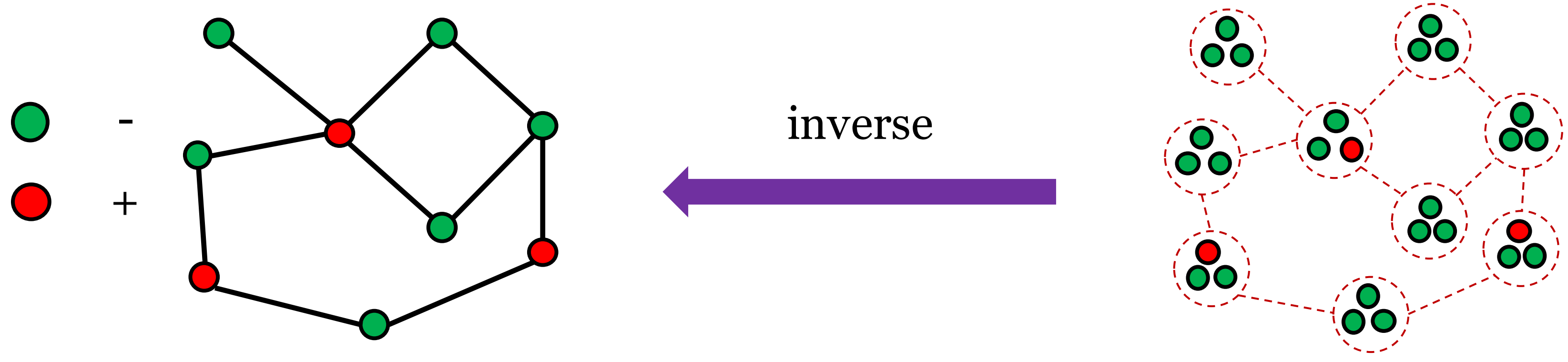
k-transformation $\longrightarrow$

Transformed distribution
μ_k over $\{0,1\}^{V_k}$
$V_k = \{u_1, u_2, \ldots, u_k \mid u \in V\}$

$X \sim \mu$ $\xleftarrow{\textbf{inverse } k\textbf{-transformation}}$ $Y \sim \mu_k$

inverse
***k*-transformation**

$X = \text{inverse}(Y)$ is uniquely fixed by Y

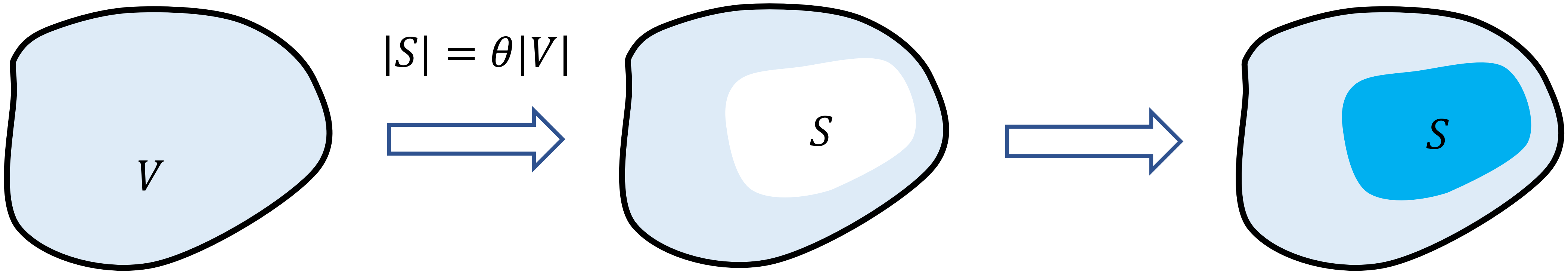


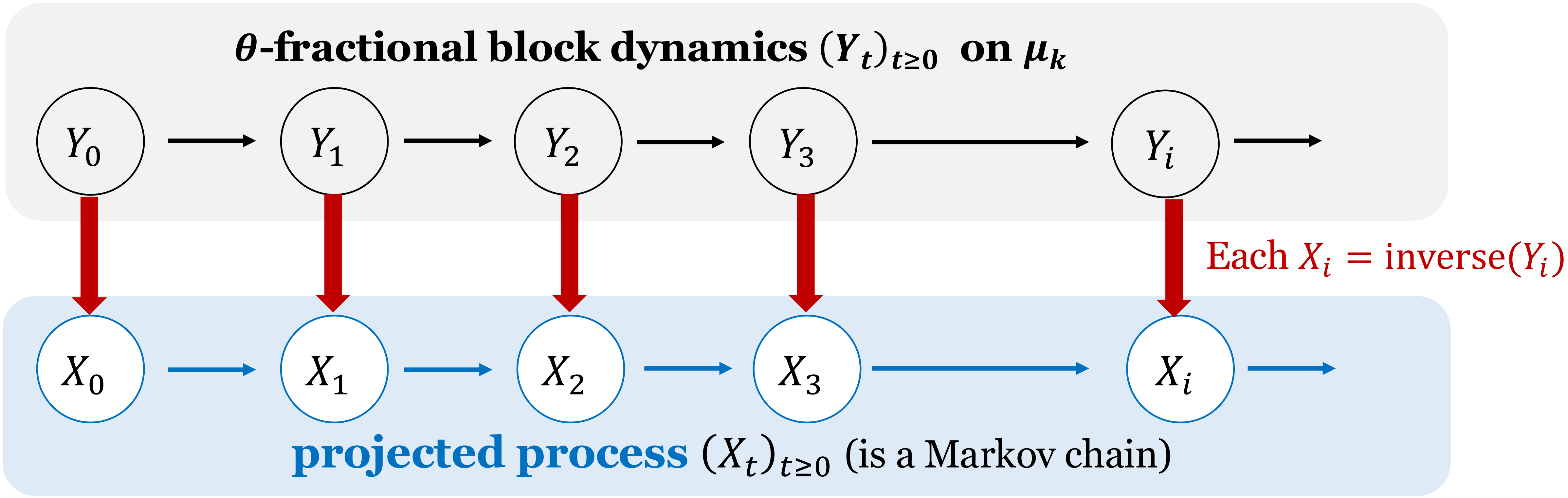
$X \sim \mu$ $Y \sim \mu_3$

θ-fractional block dynamics

Transition step: given configuration $X \in \{0,1\}^V$

- pick $S \subseteq V$ with $|S| = \theta|V|$ uniformly at random
- resample $X_S \sim \mu_S(\cdot \mid X_{V-S})$

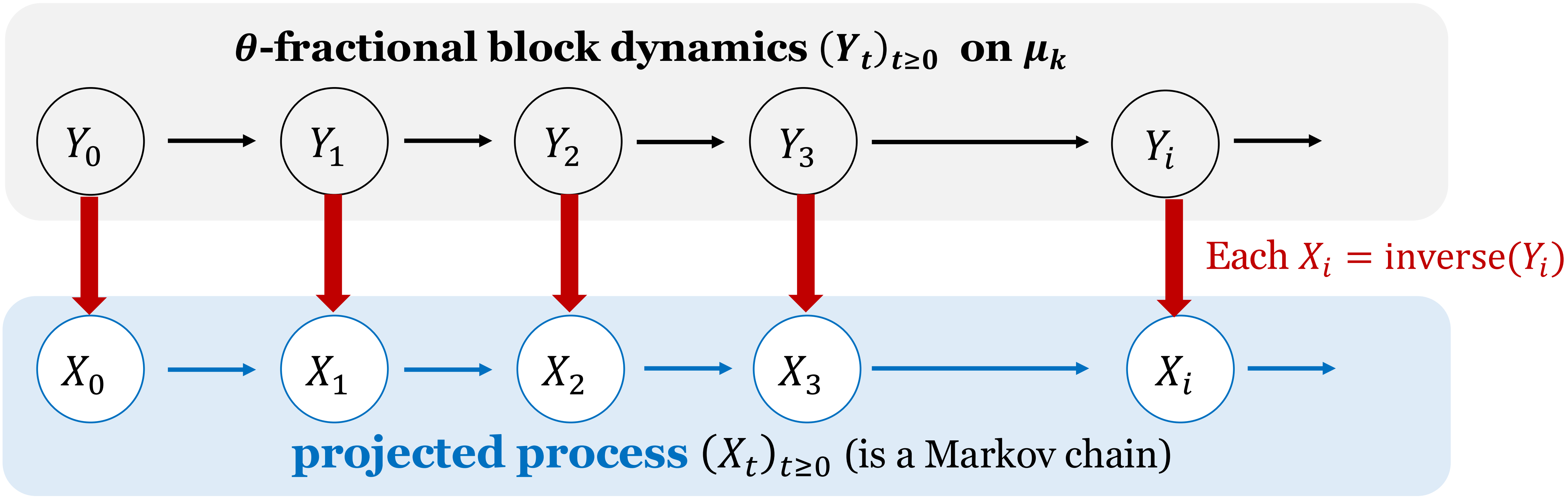




Theorem [Chen, F., Yin, Zhang, 2021]

Field dynamics is the projected process when $k \to \infty$

$$\forall \epsilon > 0, \exists K_0 \text{ s.t. } \forall k \geq K_0$$
$$\forall \sigma, \tau \in \{-,+\}^V, \left| P^{FD}(\sigma,\tau) - P^{\text{Proj}}_{\mu_k}(\sigma,\tau) \right| \leq \epsilon$$



Theorem [Chen, F., Yin, Zhang, 2021]

Field dynamics is the projected process when $k \to \infty$

Block dynamics spectral gap $\Rightarrow$ Field dynamics spectral gap

Theorem [Chen, F., Yin, Zhang, 2021]

Field dynamics is the projected process when $k \to \infty$

Block dynamics spectral gap $\Rightarrow$ Field dynamics spectral gap

Using the k-transformation, we can show that

μ is completely SI $\xrightarrow{\text{all large } k}$ **spectral independence** for distribution μ_k

Lemma: the spectral gap of ℓ-block dynamics for $\ell = \theta n$ is

$$\geq \left(\frac{\ell}{n}\right)^{O(\eta)} = \theta^{O(\eta)}$$

Boosting Spectral Gap

Field dynamics: Mixing lemma

Complete Spectral independence

field dynamics approximate tensorization of variance

Compare spectral gap:

$$\lambda_{\text{gap}}^{GD}(\mu) \geq \alpha(\mu,\theta) \cdot \lambda_{\text{mingap}}^{\text{GD}}(\boldsymbol{\theta} * \mu)$$

$O(n^2 \log n)$ mixing bound

Boosting MLS constant

Field dynamics: Mixing lemma

Complete Spectral independence
Marginal stability

field dynamics has approximate tensorization of **entropy**

Compare modified log-Sobolev constant

$$\rho_{\text{mls}}^{GD}(\mu) \geq \beta(\mu,\theta) \cdot \rho_{\text{minmls}}^{\text{GD}}(\boldsymbol{\theta} * \mu)$$

$O(n \log n)$ mixing bound

References

- **Other analysis of field dynamics**

Negative fields **localization** scheme

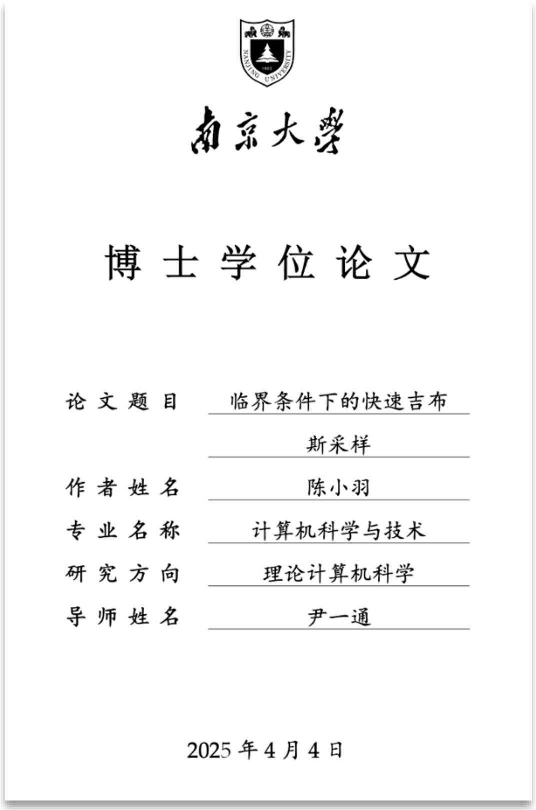
- Chen and Eldan ArXiv: 2203.04163
- Chen, Liu, and Yin ArXiv: 2305.00186

Noising and denoising Markov processes

- Chen, Chen, Yin, and Zhang ArXiv: 2411.03413

Trickle-down theorem for field dynamics

- Chen, Chen, Chen, Yin, Zhang ArXiv: 2504.03406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博士学位论文

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- **Edge-tilting field dynamics**

Chen, Ju, Miao, Yin, Zhang, ArXiv: 2604.10525